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LETTER TO THE EDITOR

Graph bipartitioning and spin glasses on a random network of fixed finite valence

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Abstract. We study the problem of bipartitioning a random graph of fixed finite valence using a mean-field replica-symmetric theory of an Ising ferromagnet with zero magnetisation constraint. The thermodynamics is determined by the probability distribution of an auxiliary field. The expression for the ground-state energy agrees with that proposed by Mézard and Parisi using a cavity-field method, but their expression for the fraction of crazy spins is reinterpreted.

Recently, techniques from the theory of spin glasses have been increasingly applied to the study of complex optimisation problems. In the so-called graph bipartitioning problem [1], we have a set of randomly connected vertices and the issue is to partition them into two subsets of equal size in such a way that the number of connections between the two sets is minimised. This problem can be mapped into that of finding the ground state of a randomly connected ferromagnetic Ising model subject to the constraint that the total magnetisation is zero [1, 2]. For the case of extensive connectivity, in which the probability p that any two sites are connected is independent of the total number of sites N, the problem is solved [1, 3] by mapping onto the infinite-range Sherrington-Kirkpatrick model of a spin glass [4, 5]. For the case of finite (but not fixed) valence [6], in which bonds between vertices are distributed with independent random probability $p = c/N$, c independent of N, the problem is equivalent to finding the ground state of the Viana-Bray model of a dilute spin glass [7, 8]. On the other hand, numerical results have been obtained for the case of finite and fixed valence, in which every site has the same coordination number c [9]. Although estimates for this case have been given by extending the theory for average valence [6] or by mapping onto the spin glass on a Bethe lattice [10], no complete theory for the fixed finite-valenced network has yet been presented.

In this letter we give a mean-field theory of the problem with fixed finite valence. In many aspects, our theory is very similar to that for the average finite valence [6, 8]. In the latter case, the ground-state properties are determined by the local field distribution, whose weighted moments give the order parameters $\{Q_k\}$ in the replica symmetric ansatz. In the case of fixed finite valence, however, we are going to show that the order parameters $\{Q_k\}$ are *not* the weighted moments of the local field h, but those of an auxiliary field Φ. This auxiliary field Φ at a site, being interpreted as the effective field due to $(c-1)$ of its c neighbouring† sites, is analogous to the effective field due

† We use the expression 'neighbour' to refer to a site/spin to which there is a single-edge connection. No implication of spatial locality is intended, or appropriate.

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to descendents on a Bethe lattice [11]. Indeed, our approach confirms that the local structure of the network is tree like, and the picture of a Bethe lattice [10] is a valid local description of the system in the thermodynamic limit. We derive an expression for the ground-state energy which agrees with a previous proposal [6], but the corresponding expression for the fraction of crazy spins has to be reinterpreted [6].

Before proceeding, it is expedient to write down the recursion relation on a Bethe lattice for later comparison [10]. The effective field Φ due to the $(c-1)$ descendents of a site obeys the distribution function $\pi(\Phi)$ given by

$$\pi(\Phi)=\prod_{i=1}^{c-1}\left(\int \mathrm{d}\Phi_i\,\pi(\Phi_i)\right)\delta\left(\Phi-\frac{1}{\beta}\sum_{i=1}^{c-1}\tanh^{-1}(\tanh\beta J\tanh\beta\Phi_i)\right) \tag{1}$$

where J is the coupling strength between neighbouring sites, and we have assumed that the sites are sufficiently distant from the boundary sites so that a fixed-point configuration has been reached. The total field h at a site then obeys the distribution function $P(h)$ given by

$$P(h)=\prod_{i=1}^{c}\left(\int \mathrm{d}\Phi_i\,\pi(\Phi_i)\right)\delta\left(h-\frac{1}{\beta}\sum_{i=1}^{c}\tanh^{-1}(\tanh\beta J\tanh\beta\Phi_i)\right). \tag{2}$$

For a symmetric distribution of fields, the fraction p_0 of crazy spins (i.e. spins located at sites having zero local field) at zero temperature should then be given by

$$p_0=\sum_{r=0}^{\mathrm{int}(c/2)}\frac{c!}{(c-2r)!(r!)^2}\pi_0^{c-2r}\left(\frac{1-\pi_0}{2}\right)^{2r} \tag{3a}$$

where π_0 is the solution of the equation

$$\pi_0=\sum_{r=0}^{\mathrm{int}((c-1)/2)}\frac{(c-1)!}{(c-1-2r)!(r!)^2}\pi_0^{c-1-2r}\left(\frac{1-\pi_0}{2}\right)^{2r}. \tag{3b}$$

As we shall see, the same relations exist in our mean-field theory, showing that the Bethe spin glass may provide a framework for interpreting the theory.

The major mathematical difficulty for a randomly connected network is that sites are not inherently equivalent. This is surpassed by considering configuration averages over all networks of fixed valence c. Thus we start by considering the Ising ferromagnet

$$H=-\sum_{(ij)}a_{ij}JS_iS_j \tag{4}$$

where $a_{ij}=0$ or 1, satisfying

$$\sum_{j\neq i}a_{ij}=c \qquad \text{for all } i. \tag{5}$$

Each set of $\{a_{ij}\}$ satisfying (5) then corresponds to a particular configuration of the network.

In the graph bipartitioning problem, each of the N Ising spins takes the value $+1$ if it belongs to one subset and -1 if it belongs to the other. The number of connections N_{ct} between the two subsets, which is the quantity to be minimised, is related to the energy E of the Ising ferromagnet via the relation

$$E=2N_{\mathrm{ct}}-\tfrac{1}{2}cN. \tag{6}$$

Since the two subsets have to be of equal size, the total magnetisation is restricted to be zero.

We shall now derive the configuration-averaged free energy of this system using the replica method [12]. This requires us to calculate the configuration average of the nth power of the partition function given by

$$\overline{Z^n} = \mathbb{N}^{-1} \sum_{a_{ij}=\{0,1\}} \prod_i \delta\left(\sum_{j\neq i} a_{ij} - c\right) \mathrm{Tr}'_\alpha \exp\left(\sum_{(ij)} \beta J a_{ij} \sum_\alpha S_i^\alpha S_j^\alpha\right) \tag{7}$$

where Tr'_α denotes the restricted trace

$$\mathrm{Tr}_\alpha \prod_\alpha \delta\left(\sum_i S_i^\alpha\right)$$

and $\mathbb{N}$ is the normalising factor, which is the total number of network configurations, i.e.

$$\mathbb{N} = \sum_{a_{ij}=\{0,1\}} \prod_i \delta\left(\sum_{j\neq i} a_{ij} - c\right). \tag{8}$$

As a first step towards evaluating these expressions, let us consider the normalising factor $\mathbb{N}$ first. Introducing the integral representation of the delta function, we have

$$\begin{aligned}\mathbb{N} &= \prod_i \left(\int_0^{2\pi} \frac{\mathrm{d}\lambda_i}{2\pi} \exp(-\mathrm{i}c\lambda_i)\right) \prod_{(ij)} (1+\exp[\mathrm{i}(\lambda_i+\lambda_j)]) \\ &= \prod_i \left(\int_0^{2\pi} \frac{\mathrm{d}\lambda_i}{2\pi} \exp(-\mathrm{i}c\lambda_i)\right) \exp\left(\sum_{(ij)} \sum_{m=1}^{\infty} \frac{(-1)^{m-1}}{m} \exp[\mathrm{i}m(\lambda_i+\lambda_j)]\right).\end{aligned} \tag{9}$$

Using Gaussian identities $\mathbb{N}$ can be further expressed in terms of a single site integral,

$$\begin{aligned}\mathbb{N} = \prod_{m=1}^{\infty} \left(\int \frac{\mathrm{d}q_m}{(2\pi m)^{1/2}} \exp(-q_m^2/2m)\right) \Bigg[\int_0^{2\pi} \frac{\mathrm{d}\lambda}{2\pi} \exp(-\mathrm{i}c\lambda) \\ \times \exp\left(\sum_{m=1}^{\infty} \frac{1}{m}[n_m q_m \exp(\mathrm{i}m\lambda) - \tfrac{1}{2}(-1)^{m-1}\exp(2\mathrm{i}m\lambda)]\right)\Bigg]^N\end{aligned} \tag{10}$$

where

$$n_m = \begin{cases} 1 & m \text{ odd} \\ i & m \text{ even.}\end{cases} \tag{11}$$

The integral over λ can be simplified by the substitution $z = \mathrm{e}^{\mathrm{i}\lambda}$, and the standard method of residues give a polynomial in terms of q_1 up to q_c. The variables from q_{c+1} upward are therefore irrelevant. The integrals over q_1 to q_c are now performed by the method of steepest descent and in the limit $N\to\infty$ the only contributing term in the polynomial is $q_1^c/c!$. Therefore

$$\begin{aligned}\mathbb{N} &= \int \frac{\mathrm{d}q_1}{(2\pi)^{1/2}} \exp(-\tfrac{1}{2}q_1^2)\left(\frac{q_1^c}{c!}\right)^N \\ &\sim \exp\{N[\tfrac{1}{2}c(\ln cN - 1) - \ln c!]\}.\end{aligned} \tag{12}$$

We can now evaluate $\overline{Z^n}$ in the same manner. From (7), we have

$$\begin{aligned}\overline{Z^n} = \mathbb{N}^{-1} \prod_i \left(\int_0^{2\pi} \frac{\mathrm{d}\lambda_i}{2\pi} \exp(-\mathrm{i}c\lambda_i)\right) \mathrm{Tr}'_\alpha \exp \sum_{(ij)} \sum_{m=1}^{\infty} \frac{(-1)^{m-1}}{m} \\ \times \exp\left[m\left(\beta J \sum_\alpha S_i^\alpha S_j^\alpha + \mathrm{i}\lambda_i + \mathrm{i}\lambda_j\right)\right].\end{aligned} \tag{13}$$

Expressing the term involving spin variables as

$$\exp\left(m\beta J\sum_{\alpha} S_i^{\alpha} S_j^{\alpha}\right)=\prod_{\alpha}\cosh m\beta J(1+S_i^{\alpha}S_j^{\alpha}\tanh m\beta J) \tag{14}$$

and going through the steps as above, we arrive at

$$\begin{aligned}\overline{Z^n}=\mathbb{N}^{-1}\prod_{\alpha}&\left(\int\frac{\mathrm{d}x^{\alpha}}{(2\pi/\beta J_1)^{1/2}}\exp(-\tfrac{1}{2}\beta J_1(x^{\alpha})^2)\right)\\&\times\prod_{m=1}^{\infty}\left\{\left[\int\frac{\mathrm{d}q_m^0}{(2\pi m/\cosh^n m\beta J)^{1/2}}\right.\right.\\&\left.\times\exp\left(-\frac{\cosh^n m\beta J}{2m}(q_m^{\alpha})^2\right)\right]\\&\times\prod_{\alpha}\left[\int\frac{\mathrm{d}q_m^{\alpha}}{(2\pi m/\cosh^n m\beta J\tanh m\beta J)^{1/2}}\right.\\&\left.\times\exp\left(-\frac{\cosh^n m\beta J\tanh m\beta J}{2m}(q_m^{\alpha})^2\right)\right]\\&\times\prod_{\alpha<\beta}\left[\int\frac{\mathrm{d}q_m^{\alpha\beta}}{(2\pi m/\cosh^n m\beta J\tanh^2 m\beta J)^{1/2}}\right.\\&\left.\left.\times\exp\left(-\frac{\cosh^n m\beta J\tanh^2 m\beta J}{2m}(q_m^{\alpha\beta})^2\right)\right]\dots\right\}\\&\times\left\{\mathrm{Tr}_{\alpha}\int_0^{2\pi}\frac{\mathrm{d}\lambda}{2\pi}\exp(-\mathrm{i}c\lambda)\exp\left[\mathrm{i}\beta J_1\sum_{\alpha}x^{\alpha}S^{\alpha}\right.\right.\\&+\sum_{m=1}^{\infty}\frac{\cosh^n m\beta J}{m}n_m\exp(\mathrm{i}m\lambda)\left(q_m^0+\tanh m\beta J\sum_{\alpha}q_m^{\alpha}S^{\alpha}\right.\\&\left.\left.\left.+\tanh^2 m\beta J\sum_{\alpha<\beta}q_m^{\alpha\beta}S^{\alpha}S^{\beta}+\dots\right)-\frac{(-1)^{m-1}}{2m}(\exp(m\beta J))^n\exp(2\mathrm{i}m\lambda)\right]\right\}^N\end{aligned} \tag{15}$$

where we have replaced the restricted trace Tr'_{α} by a global soft constraint term $\frac{1}{2}J_1(\Sigma_i S_i)^2$ in the Hamiltonian [1]. When compared with the case for average finite valence [8], each order parameter is further decomposed into components indexed by m. Fortunately, following the text discussion below (11), only the $m=1$ components are relevant in the thermodynamic limit. This permits us to write

$$\begin{aligned}\overline{Z^n}=\mathbb{N}^{-1}\prod_{\alpha}&\left(\int\frac{\mathrm{d}x^{\alpha}}{(2\pi/\beta J_1)^{1/2}}\right)\int\frac{\mathrm{d}q_1^0}{(2\pi/\cosh^n\beta J)^{1/2}}\\&\times\prod_{\alpha}\int\frac{\mathrm{d}q_1^{\alpha}}{(2\pi/\cosh^n\beta J\tanh\beta J)^{1/2}}\prod_{\alpha<\beta}\int\frac{\mathrm{d}q_1^{\alpha\beta}}{(2\pi/\cosh^n\beta J\tanh^2\beta J)^{1/2}}\cdots\\&\times\exp\left[-\tfrac{1}{2}\beta J_1(x^{\alpha})^2-\tfrac{1}{2}\cosh^n\beta J\left((q_1^0)^2+\tanh\beta J\sum_{\alpha}(q_1^{\alpha})^2\right.\right.\\&\left.\left.+\tanh^2\beta J\sum_{\alpha<\beta}(q_1^{\alpha\beta})^2+\dots\right)+N\ln\mathrm{Tr}_{\alpha}X\right]\end{aligned} \tag{16a}$$

where

$$X=\frac{1}{c!}\left[\exp\left(\mathrm{i}\beta J_1\sum_\alpha x^\alpha S^\alpha\right)\right]\left[\cosh^n\beta J\left(q_1^0+\tanh\beta J\sum_\alpha q_1^\alpha S^\alpha\right.\right.$$
$$\left.\left.+\tanh^2\beta J\sum_{\alpha<\beta}q_1^{\alpha\beta}S^\alpha S^\beta+\ldots\right)\right]^c. \tag{16b}$$

We note in passing that, apart from normalising factors, (16b) corresponds to the cth term of the power series expansion in the average finite-valence expression [8].

The integrals are now evaluated using the method of steepest descent. At the extremum of the integrand, we have

$$\begin{aligned}
x^\mu &= \mathrm{i}N\langle S^\mu\rangle\\
q_1^0 &= \frac{cN}{\cosh^n\beta J}\left\langle\left(q_1^0+\tanh\beta J\sum_\alpha q_1^\alpha S^\alpha+\ldots\right)^{-1}\right\rangle\\
q_1^\mu &= \frac{cN}{\cosh^n\beta J}\left\langle S^\mu\left(q_1^0+\tanh\beta J\sum_\alpha q_1^\alpha S^\alpha+\ldots\right)^{-1}\right\rangle\\
q_1^{\mu\nu} &= \frac{cN}{\cosh^n\beta J}\left\langle S^\mu S^\nu\left(q_1^0+\tanh\beta J\sum_\alpha q_1^\alpha S^\alpha+\ldots\right)^{-1}\right\rangle
\end{aligned} \tag{17}$$

etc, where

$$\langle A\rangle=\mathrm{Tr}_\alpha AX/\mathrm{Tr}_\alpha X. \tag{18}$$

Already we note an important difference from the Sherrington–Kirkpatrick [4] and Viana–Bray [7] spin glasses. In those models, the order parameters are the unweighted thermodynamic averages of the spin variables, whereas in our case they are the thermodynamic averages of the spin variables weighted by the factor $(q_1^0+\tanh\beta J\,\Sigma_\alpha\, q_1^\alpha S^\alpha+\ldots)^{-1}$. It is this difference that eventually causes the ground-state properties to be determined by the auxiliary field distribution, in contrast to the dependence on the local field distribution in the average valence case.

We now solve the problem within the framework of the replica-symmetric ansatz. The order parameters in (17) are assumed to be independent of the replica indices, so that

$$\begin{aligned}
x^\mu &= x\\
q_1^0 &= \left(\frac{cN}{\cosh^n\beta J}\right)^{1/2}Q_0\\
q_1^\mu &= \left(\frac{cN}{\cosh^n\beta J}\right)^{1/2}Q_1\\
q_1^{\mu\nu} &= \left(\frac{cN}{\cosh^n\beta J}\right)^{1/2}Q_2
\end{aligned} \tag{19}$$

etc. Since the Q_k are the weighted averages of the spin variables, it is natural to introduce the auxiliary field distribution $\pi(\Phi)$[6,8,12]† according to

$$Q_k=\int \mathrm{d}\Phi\;\pi(\Phi)\tanh^k\beta\Phi. \tag{20}$$

† An alternative procedure would be to use a global order parameter function, such as introduced by De Dominicis and Mottishaw [13].

This facilitates a simplification of the extremum conditions (17), which become, in the limit $n \to 0$,

$$\frac{x}{\mathrm{i}N} = \prod_{i=1}^{c} \left(\int \mathrm{d}\Phi_i\, \pi(\Phi_i) \right) \tanh\left(\mathrm{i}\beta J_1 x + \sum_{i=1}^{c} \tanh^{-1}(\tanh \beta J \tanh \beta\Phi_i) \right) \tag{21a}$$

$$Q_k = \prod_{i=1}^{c-1} \left(\int \mathrm{d}\Phi_i\, \pi(\Phi_i) \right) \tanh^k\left(\mathrm{i}\beta J_1 x + \sum_{i=1}^{c-1} \tanh^{-1}(\tanh \beta J \tanh \beta\Phi_i) \right). \tag{21b}$$

Although we do not solve the equation for x in full generality, it is sufficient to note that $x = 0$, together with $\pi(\Phi)$ an even function of Φ, satisfies equations (20) and (21). This is in accordance with the argument that the zero-magnetisation constraint is irrelevant to the ground state of the spin glass [1]. Combining (20) and (21b) permits us to write

$$\pi(\Phi) = \prod_{i=1}^{c-1} \left(\int \mathrm{d}\Phi_i\, \pi(\Phi_i) \right) \delta\left(\Phi - \frac{1}{\beta} \sum_{i=1}^{c-1} \tanh^{-1}(\tanh \beta J \tanh \beta\Phi_i) \right). \tag{22}$$

Note the formal identity between (22) and (1). This shows that the auxiliary field Φ is the equivalent of the field due to descendents in the Bethe lattice.

It is instructive to determine the relation between the auxiliary field distribution $\pi(\Phi)$ and the local field distribution $P(h)$, which is related to the thermodynamic average of spins by

$$\langle S^{\mu_1} \ldots S^{\mu_k} \rangle = \int \mathrm{d}h\, P(h) \tanh^k \beta h. \tag{23}$$

Performing the thermodynamic average explicitly, we finally arrive at

$$P(h) = \prod_{i=1}^{c} \left(\int \mathrm{d}\Phi_i\, \pi(\Phi_i) \right) \delta\left(h - \frac{1}{\beta} \sum_{i=1}^{c} \tanh^{-1}(\tanh \beta J \tanh \beta\Phi_i) \right). \tag{24}$$

Again, note the formal identity between (24) and (2), confirming the validity of the picture of a tree-like structure in the random network.

At $T = 0$, the probability π_0 that the auxiliary field is zero satisfies (3b) and is given in [6, 10]. Note, however, that this probability is distinct from p_0 given in (3a), which is the probability that the local field is zero or, equivalently, the probability of a spin being crazy. Thus, for instance, $\Phi = 0$ does not necessarily imply an ill defined spin, but $\Phi = \pm J$ may result in one. Both π_0 and p_0 are given in table 1 for $3 \leq c \leq 8$.

Finally, we evaluate the free energy per site according to the replica theory [14]:

$$-\beta f = \lim_{N \to \infty} \lim_{n \to 0} \frac{\overline{Z^n} - 1}{Nn}. \tag{25}$$

Table 1. Table of zero auxiliary field distribution (π_0), zero local field distribution (p_0) and ground-state energy for $3 \leq c \leq 8$.

	c					
	3	4	5	6	7	8
π_0	0.333	0.200	0.229	0.167	0.183	0.146
p_0	0.259	0.232	0.188	0.184	0.156	0.156
$-E_0/N_b J$	0.852	0.744	0.676	0.619	0.580	0.540

After some algebra, we obtain

$$-\beta f = \tfrac{1}{2}c \ln \cosh \beta J + \tfrac{1}{2}c \int \mathrm{d}\Phi\, \pi(\Phi) \ln(1 - \tanh^2 \beta J \tanh^2 \beta\Phi)$$
$$- \tfrac{1}{2}c \int \mathrm{d}\Phi_1\, \mathrm{d}\Phi_2\, \pi(\Phi_1)\pi(\Phi_2) \ln(1 + \tanh \beta J \tanh \beta\Phi_1 \tanh \beta\Phi_2)$$
$$+ \int \mathrm{d}h\, P(h) \ln 2 \cosh \beta h \tag{26}$$

and the ground-state energy per site is found to be

$$-\frac{E_0}{NJ} = \tfrac{1}{2}c\pi(0)^2 + 2 \sum_{r=1}^{c} P(r) r. \tag{27}$$

This formula can be shown to be equivalent to that proposed by Mézard and Parisi [6]. The ground-state energies per bond for $3 \leqslant c \leqslant 8$ are listed in table 1, and are of the order of 1-3% lower than the simulation results [9].

We have derived the above result for an Ising ferromagnet with a zero-magnetisation constraint. The generalisation to an exchange-random Ising spin glass is straightforward: the corresponding values for $\pi(\Phi)$, $P(h)$ and $-\beta f$ are obtained by taking the disorder average over J in equations (22), (24) and (27), respectively.

In summary, we have studied the graph bipartitioning and spin glasses on a randomly connected network of fixed finite valence. We find that the ground-state properties are determined by the probability distribution of the auxiliary field, confirming the picture of the Bethe lattice on the network. We have also developed techniques for the study of fixed-valence networks, and it is hoped that the study of other issues such as replica symmetry breaking on these networks will further our understanding of the graph bipartitioning problem and spin glasses.

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Note added in proof. In equations ($3a$), ($3b$) and (27), we have assumed that Φ and h are integral multiples of J. It has been drawn to our attention by D J Thouless and P Mottishaw that such solutions are unstable. The stable solutions have a continuous distribution, but numerical investigation, to be reported elsewhere, has shown that the ground-state energies are only slightly altered.

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